Behavior of Liquid Surface Impinged by Turbulent Jets

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The effect of fluid jet interaction with a free liquid surface has been the subject of many experimental and theoretical studies because of numerous potential applicabilities of such a system. Steelmaking and drying, absorption, and mixing of jets with an ambient liquid are the most relevant examples. Apart from the maximum depth of jet penetration into the free liquid surface, cavity shape and size, splashing, and mixing process in the receiving liquid have also been extensively studied. The majority of the papers deal with gaseous jets impinging downward onto the free liquid surface. However, several papers have also been published on liquid-submerged jets acting on the free surface directed upward (Ackerman et al., 1971; Banks and Bhavamai, 1965; Hunt and Hsu, 1965; Murota and Muraoka, 1967; Narusawa et al., 1983). It is interesting to make a more comprehensive comparison of the data and theories developed so far for the various systems in order to establish more general conclusions that may be useful for designing purposes.

The theoretical approaches valid for turbulent, free jets are based on the relationships describing behavior of such jets, i.e., velocity distribution along the jet axis, stagnation pressure analysis, displaced liquid analysis, and application of the entrainment coefficients (Ackerman et al., 1971; Banks et al., 1963, 1965; Chatterjee and Bradshaw, 1972; Cheslak et al., 1969; Davenport et al., 1966; Ito et al., 1976; Lohe, 1966; Narusawa et al., 1983; Struck, 1965; Turkdogan, 1966).

If we neglect surface tension and shear on the cavity (or bulge) wall, the jet penetration depth into the liquid surface is determined by the dynamic pressure associated with the centerline velocity of a free, turbulent, incompressible jet. The centerline velocity of the turbulent jet is given by

$$\frac{V_x}{V_0} = K\left(\frac{d_0}{x}\right) \tag{1}$$

where K is an empirically determined constant, corresponding to the potential core length of the jet. From the Bernoulli equation

$$\frac{1}{2}\rho_j V_x^2 = \gamma_L h_m \tag{2}$$

For deep cavities (higher bulges) $x = h_m + H$ so that from Eqs. 1 and 2

$$h_m(h_m + H)^2 = \frac{1}{2}\rho_j K^2 d_0^2 V_0^2/\gamma_L \tag{3}$$

or

$$(h_m/H)(h_m/H + 1)^2 = \frac{2K^2}{\pi} m_H \tag{4}$$

where m_H is a dimensionless momentum of the jet based on H.

The displaced liquid analysis assumes that the force the jet exerts on the liquid surface is equal to the weight of the displaced liquid. It also assumes that the depression (or the bulge) is sufficiently small to not appreciably alter the velocity and pressure distribution of the jet flow. This implies that the vertical component of momentum of the departing jet flow is zero. The weight of the displaced liquid can be determined from the assumed cavity profile that is established by a known pressure distribution on the surface. The analysis given by Banks and Chandrasekhara (1963) shows that

$$\frac{h_m}{H} = \frac{(\pi/\beta_*)^{1/2}}{\left(\frac{M}{\gamma_L h_m^3}\right)^{1/2} - (\pi/\beta_*)^{1/2}} \tag{5}$$

or

$$(h_m/H)(h_m/H + 1)^2 = (\beta_*/\pi)m_H \tag{6}$$

where β_* is a constant in an equation approximating the cavity (bulge) shape

$$h/h_m = \exp[-\beta_*(r/H)^2] \tag{7}$$

Comparison of Eqs. 4 and 6 indicates that $\beta_* = 2K^2$. Equation 7 has the same form as that for the pressure distribution of the jet impinging on the flat surface. Table 1 summarizes the values of the constant terms $2K^2/\pi$ or β_*/π given by different authors. The range of the values of the dimensionless momentum, m_H, and the systems studied are also indicated in the table. From the values of the constant terms shown in Table 1 one can conclude that the average value of about 40 can be accepted with the exception of the data of Hunt and Hsu (1965).

For shallow cavities (small bulges), i.e., when $h_m \ll H$, Eqs. 4 and 6 simplify into the relationships yielding simple proportionality between h_m/H and m_H.

Some authors (Ackerman et al., 1971; Banks et al., 1963, 1965) accounted for the effect of surface tension. Thus a correction factor, $(1 + 4\beta_*\sigma/\Delta\gamma H^2)^{-1}$, should be introduced into Eq. 4 or 6 by multiplying their right-hand side by this factor. However, the remaining authors neglected the effect of surface tension, and the data collected by them for a wide varity of liquids with different surface tensions seem to confirm this conclusion.

The experiments carried out by different authors, as it is demonstrated in Table 1, cover a wide range of fluid jet physical parameters, as well as those for the receiving liquid. Data due to different authors, mainly those that have not been correlated by means of the product $(h_m/H)(h_m/H + 1)^2$, are shown in Figure

TABLE 1. VALUES OF $2K^2/\pi$ OR β_*/π IN EQ. 4 OR 6

Systems Jet	Studied Liquid	$2K^2/\pi$ or β_*/π	Range of m_H Values	References
Water	Water	45 ± 11	0.00015–0.0072	Ackerman et al. (1971)
Air	Water			
Oil	Water	40 ± 8	0.0008–120	Banks and Bhavamai (1965)
Water	CCl_4			
Air	Water	40 ± 8	0.0003–0.0030	Banks and Chandrasekhara (1963)
He, N_2, CO_2, H_2, Air, SO_2	Water, organic liquid, molten metals	37 ± 6	0.0003–0.070	Chatterjee and Bradshaw (1972)
Air, He	Water, cement, Hg, organic liquids	44 ± 10	0.0007–0.10	Cheslak et al. (1969)
CO_2	Water, 4% PVA solution	37 ± 6	0.0002–0.025	Davenport et al. (1966)
Water	Water	24.5[a], 29–37[b]	0.0012–0.06	Hunt and Hsu (1965)
Air	Water, organic liquid	34 ± 7	0.001–0.10	Ito et al. (1976)
Air	Water	39 ± 6		Struck (1965)
Air	Water, $Pb(ClO_4)_2$	44 ± 5	0.00015–0.012	Turkdogan (1966)

[a] As given by the authors.
[b] From experimental data for $H/d_0 \geq 9$.

1. A straight line corresponding to Eq. 4 or 6 with a constant term equal to 40 is also drawn in this figure. It can be concluded that the form of the theoretical Eq. 4 or 6 is confirmed by the trend of the experimental data points with the exception of the data of Shrivastava et al. (1976), which lie somewhat higher than the rest of the data points. Figure 1 does not contain the larger numbers of

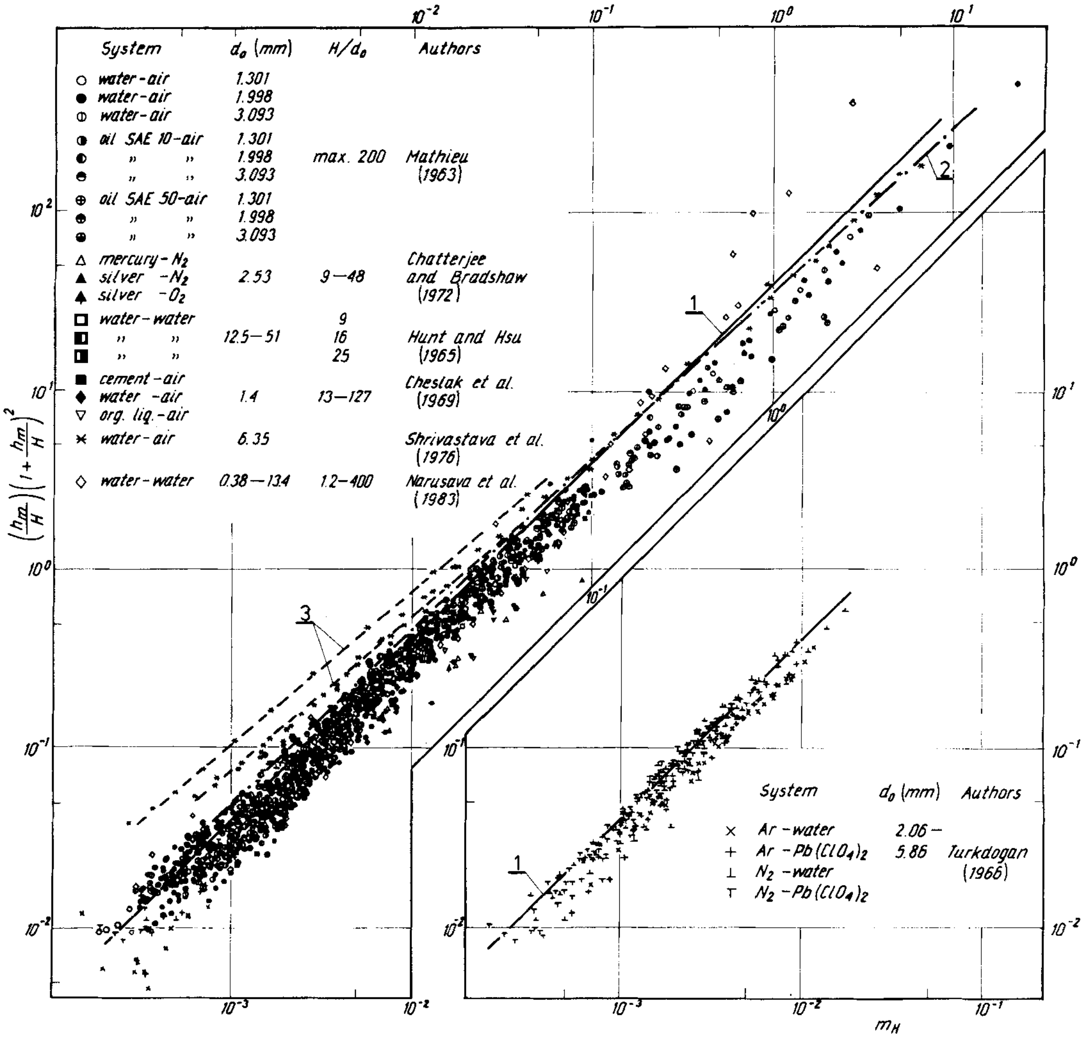

Figure 1. Experimental data of different authors: $(h_m/H)(h_m/H+1)^2$ vs. m_H: 1—Eq. 4 or 6 with $2K^2/\pi = \beta_*/\pi = 40$; 2—according to Collins and Lubanska (1954); 3—approximation of the data of Shrivastava et al. (1976).

data points of Chatterjee and Bradshaw (1972), Ackerman et al. (1971), Banks et al. (1963, 1965), Davenport et al. (1966), or Ito et al. (1976), since they are well represented by Eq. 4 or 6 with the values of the constant terms as given in Table 1.

Collins and Lubanska (1954) correlated their data by a different equation, based on dimensional arguments. Their data are scattered around their proposed correlation. Murota and Muraoka (1967) suggested their own correlation. This can be transformed into a form that shows $(h_m/H) \propto (H/d_0)^{1/2}\, m_H{}^{3/4}$. Lohe (1966) correlated the ratio of the cavity depth and the cavity diameter with some characteristic features of the impinging, turbulent jet. Struck (1965) and Yablonsky et al. (1974) actually gave correlations that are almost identical with Eq. 4 or 6, accounting additionally for some supersonic effects in their jets.

Narusawa et al. (1983) based their considerations on dimensional arguments and on the entrainment coefficient of the jet, β. They used some values of this coefficient reported by some earlier authors. More recent data (Abraham, 1965; Ablertson et al., 1948; Davenport et al., 1966; Fischer et al., 1979; Gassmann, 1980; Hill, 1972; Lehrer, 1981; Sforza and Mons, 1978) show that the values of the entrainment coefficient are dependent on the H/d_0 ratio and for $H/d_0 = 1$ they drop to 0.12 (potential core region of the jet). Thus β assumes a constant value, which would be closer to 0.32, only for a developed turbulent jet. Narusawa et al. (1983) applied values of H/d_0 in the range from 1.2 to 400, so that for the data with $H/d_0 < 8$ the entrainment coefficient should be taken as considerably lower than 0.45 or 0.32.

It should be pointed out that the application of the turbulent jets theory, Eq. 1, is justified only for fully developed turbulent jets. This means that the theory is applicable only if H/d_0 is greater than about 10, the exact value being somewhat dependent on the nozzle design and the jet Reynolds number (Hrycak et al., 1970). This explains to some extent some differences among the various results for shorter jets and also affects the values of the constant K. This constant may vary between 5.4 and 9.4 for the impinging jets. Most frequently, values close to 6.4 are reported (Hrycak et al., 1970; Beltaos and Rajaratnam, 1974; Kiser, 1963; Rosler and Bankoff, 1963). However, for such a value of K the constant term in Eq. 4 would be too low for the most cases. Similarly, the values of β_* in Eq. 7 may differ somewhat from that assumed by Banks et al. (1963, 1965); e.g., Beltaos and Rajaratnam (1974) reported $\beta_* = 114$ for their data on pressure distribution for impinging, turbulent jets.

The case of $H/d_0 = 0$ is of some interest. Collins and Lubanska (1954), Struck (1965), and Yablonsky et al. (1974) gave some data for this particular case. According to the recent authors the data for $H/d_0 = 0$ can be correlated as follows:

$$h_m/d_0 = \frac{1}{2} n \rho_j V_0^2 / \gamma_L d_0 \tag{8}$$

or

$$h_m/d_0 = \frac{2n}{\pi} \frac{M}{\gamma_L d_0^3} = \frac{2n}{\pi} m_d \tag{9}$$

where n expresses the ratio of the hydrostatic head for the cavity and the jet kinetic energy and is given graphically by the authors.

As the final conclusion it can be stated that there is no meaningful difference in the data collected for gaseous and liquid turbulent jets affecting a free liquid surface. The theories briefly considered in the paper proved to be approximately valid for description of the experimental data for different systems studied, provided the fully turbulent, developed jets are used ($H/d_0 > 8$). The expected accuracy of the predictions from Eq. 4 or 6 with the constant terms equal to 40 is about ±20%. Greater discrepancies may occur for shorter jets.

NOTATION

d_0 = nozzle diameter
h = height of a free surface depression (or rise)
h_m = maximum height of a free surface depression or rise
H = distance between the nozzle and a quiescent free surface
K = constant in Eq. 1
m_d = dimensionless jet momentum based on d_0, $m_d = M/\gamma_L d_0^3$
m_H = dimensionless jet momentum based on H, $m_H = M/\gamma_L H^3$
M = jet momentum, $M = \frac{1}{4}\pi d_0^2 V_0^2 \rho_j$
n = coefficient in Eqs. 8 and 9
r = cavity (bulge) radius
V_0 = jet velocity at the nozzle outlet
V_x = average jet velocity at the x distance from the nozzle
x = axial distance from the nozzle

Greek Letters

β = entrainment coefficient of jet
β_* = constant in Eqs. 5 to 7
γ_L = specific weight of the liquid
ρ_j = density of jet
σ = surface tension

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